Math 4 Honors
Implicit Differentiation Madness!

Name _____
Date _____

For each of the following equations, find $\frac{dy}{dx}$ using **implicit differentiation**.

1. $x^2 - 5xy + 3y^2 = 7$

$D(x^2) - D(5xy) + D(3y^2) = D(7)$

$$\begin{aligned} 2x - (5x\frac{dy}{dx} + 5y) + 6y\frac{dy}{dx} &= 0 \\ 2x - 5x\frac{dy}{dx} - 5y + 6y\frac{dy}{dx} &= 0 \\ \frac{dy}{dx}(-5x + 6y) &= -2x + 5y \\ \frac{dy}{dx} &= \frac{-2x + 5y}{-5x + 6y} \\ \frac{dy}{dx} &= \frac{2x - 5y}{5x - 6y} \end{aligned}$$

2. $4x^2 - 9y^2 = 17$

$D(4x^2) - D(9y^2) = D(17)$

$$\begin{aligned} 8x - 18y\frac{dy}{dx} &= 0 \\ -18y\frac{dy}{dx} &= -8x \\ \frac{dy}{dx} &= \frac{8x}{18y} \\ \frac{dy}{dx} &= \frac{4x}{9y} \end{aligned}$$

3. Find the slope of $xy^2 + x = 1$ at $\left(\frac{1}{2}, 1\right)$

$x \cdot y^2 = 1 - x$

$x \cdot 2y\frac{dy}{dx} + y^2 = -1$

$\frac{dy}{dx} = \frac{-1 - y^2}{2xy}$

$\left.\frac{dy}{dx}\right|_{(\frac{1}{2},1)} = \frac{-1 - (1)^2}{2(\frac{1}{2})(1)}$

$= \frac{-2}{1}$

$= -2$

Challenge:

4. $\cos(x+y)+\sin(x+y)=\frac{1}{3}$

$$D(\cos(x+y))+D(\sin(x+y))=D(\frac{1}{3})$$

$$\begin{aligned}
-\sin(x+y)(1+\frac{dy}{dx})+\cos(x+y)(1+\frac{dy}{dx}) &= 0\\
-\sin(x+y)-\sin(x+y)(\frac{dy}{dx})+\cos(x+y)+\cos(x+y)(\frac{dy}{dx}) &= 0\\
(\frac{dy}{dx}(\cos(x+y)-\sin(x+y)) &= \sin(x+y)-\cos(x+y)\\
\frac{dy}{dx} &= \frac{\sin(x+y)-\cos(x+y)}{\cos(x+y)-\sin(x+y)}\\
\frac{dy}{dx} &= -1
\end{aligned}$$

AP Calculus AB Exam Practice

18. If $\ln(2x + y) = x + 1$, then $\frac{dy}{dx} =$

(A) -2 (B) $2x + y - 2$ (C) $2x + y$ (D) $4x + 2y - 2$ (E) $y - \frac{y}{x}$

$$\frac{d}{dx}\left(\ln(2x+y)\right) = \frac{d}{dx}(x+1)$$

$$\frac{1}{2x+y}\cdot\left(2+\frac{dy}{dx}\right) = 1$$

$$2+\frac{dy}{dx} = 2x+y$$

$$\frac{dy}{dx} = 2x+y-2$$

27. If $(x + 2y)\cdot\frac{dy}{dx} = 2x - y$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 0)$?

Just means 2nd derivative!

(A) $-\frac{10}{3}$ (B) 0 (C) 2 (D) $\frac{10}{3}$ (E) Undefined

$$\frac{dy}{dx} = \frac{2x-y}{x+2y} \Rightarrow \left.\frac{dy}{dx}\right|_{(3,0)} = \frac{2\cdot 3-0}{3+0} = \frac{6}{3} = 2 \star$$

$$\frac{d^2y}{dx^2} = \frac{\left(2-\frac{dy}{dx}\right)(x+2y) - \left(1+2\frac{dy}{dx}\right)(2x-y)}{(x+2y)^2}$$

sub in for $\frac{dy}{dx}$

$$\left.\frac{d^2y}{dx^2}\right|_{(3,0)} = \frac{(2-2)(3+0) - (1+2\cdot 2)(2\cdot 3-0)}{(3+2\cdot 0)^2}$$

$$= \frac{0-(30)}{9}$$

$$= -\frac{30}{9}$$

$$= -\frac{10}{3}$$